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Causal diagrams and the identification of causal effects

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Overview

Causal diagrams

Interventions in Markovian models

Confounding bias

Intervention calculus

Identifiability tests

An example from Transport domain

Causal diagrams – Causal effect

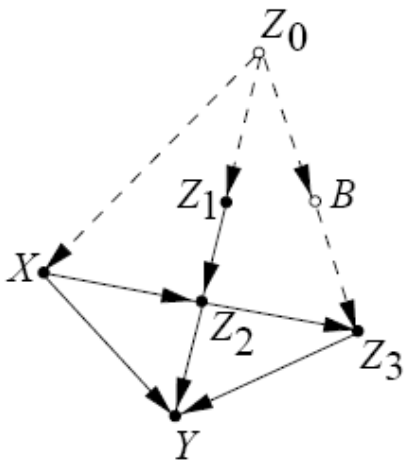


Figure 3.1: A causal diagram representing the effect of fumigants (X) on yields (Y).

$$\begin{aligned} Z_0 &= f_0(\epsilon_0), & B &= f_B(Z_0, \epsilon_B), \\ Z_1 &= f_1(Z_0, \epsilon_1), & X &= f_X(Z_0, \epsilon_X), \\ Z_2 &= f_2(X, Z_1, \epsilon_2), & Y &= f_Y(X, Z_2, Z_3, \epsilon_Y), \\ Z_3 &= f_3(B, Z_2, \epsilon_3). \end{aligned} \tag{3.3}$$

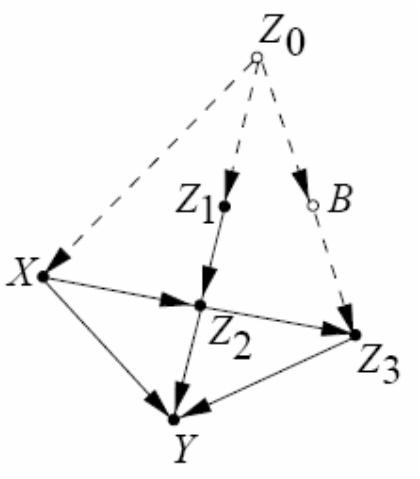
$$P(x_1, ..., x_n) = \prod_i P(x_i \mid pa_i), \tag{3.5}$$

$$\begin{aligned} P(z_0, x, z_1, b, z_2, z_3, y) &= P(z_0)P(x|z_0)P(z_1|z_0) \\ &\quad \times P(b|z_0)P(z_2|x, z_1) \\ &\quad \times P(z_3|z_2, b)P(y|x, z_2, z_3). \end{aligned} \tag{3.6}$$

Causal effect

Given two disjoint sets of variables, X and Y, the **causal effect** of X on Y, denoted either as $P(y|\hat{x})$ or as $P(y|do(x))$, is a function from X to the space of probability distributions on Y.

For each realization x of X, $P(y|\hat{x})$ gives the probability of $Y = y$ induced by deleting from the model of (3.4) all equations corresponding to variables in X and substituting $X = x$ in the remaining equations.



$$P(y \mid x^{\wedge}) = \sum_{Z_1} \sum_{Z_2} \sum_{Z_3} P(y \mid z_2, z_3, x)\ P(z_2 \mid z_1, x)\ \times \sum_{x'} P(z_3 \mid z_1, z_2, x')\ P(z_1, x')$$

Interventions as variables

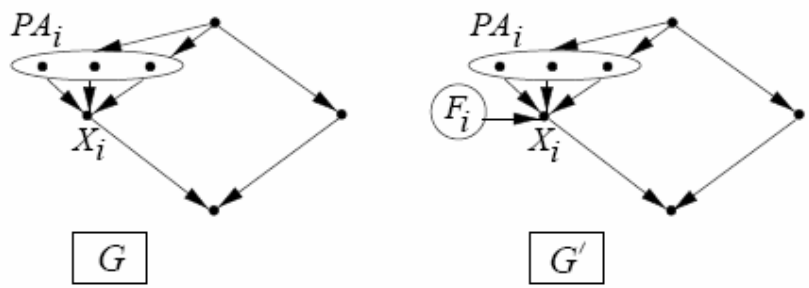


Figure 3.2: Representing external intervention F_i by an augmented network $G' = G \cup \{F_i \to X_i\}$.

$$P(x_i \mid pa_i') = \begin{cases} P(x_i \mid pa_i) & \text{if } F_i = \text{idle}, \\ 0 & \text{if } F_i = do(x_i') \\ & \text{and } x_i \neq x_i', \\ 1 & \text{if } F_i = do(x_i') \\ & \text{and } x_i = x_i'. \end{cases}$$

$$P(x_1, ..., x_n | \hat{x}_i') = P'(x_1, ..., x_n \mid F_i = do(x_i'))$$

where P' is represented by G'

Computing the effect of interventions

Truncated factorization formula

$$P(x_1, ..., x_n | \widehat{x}_i') = \begin{cases} \prod_{j \neq i} P(x_j | pa_j) & \text{if} \quad x_i = x_i', \\ 0 & \text{if} \quad x_i \neq x_i'. \end{cases}$$

$$P(x_1, ..., x_n) = \prod_i P(x_i \mid pa_i),$$

$$P(x_1, \ldots, x_n | \widehat{x}_i') = \begin{cases} \dfrac{P(x_1, \ldots, x_n)}{P(x_i' \mid pa_i)} & \text{if} \quad x_i = x_i', \\ 0 & \text{if} \quad x_i \neq x_i'. \end{cases}$$

$$P(x_1, \ldots, x_n | \widehat{x}_i') = \begin{cases} P(x_1, \ldots, x_n | x_i', pa_i) P(pa_i) & \\ & \text{if} \quad x_i = x_i', \\ 0 & \text{if} \quad x_i \neq x_i'. \end{cases}$$

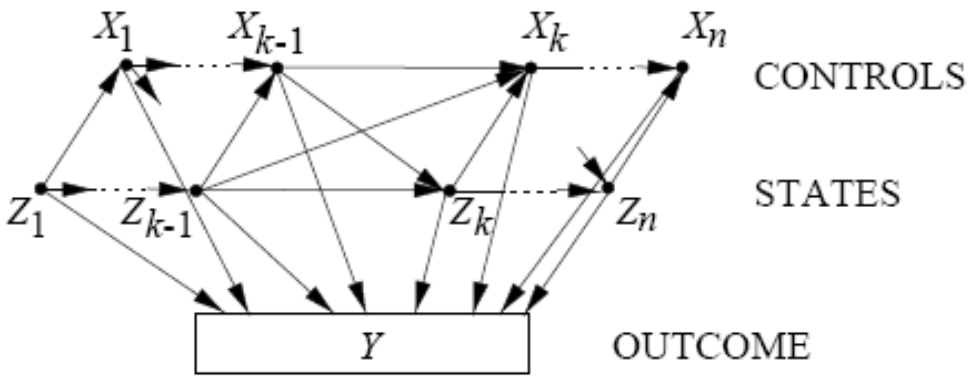
Adjustments for direct cause

Let PA_i denote the set of direct causes of variable X_i, and let Y be any set of variables disjoint of $\{X_i \cup PA_i\}$. The effect of the intervention $do(X_i = x_i')$ on Y is given by

$$P(y|\hat{x}_i') = \sum_{pa_i} P(y|x_i', pa_i)P(pa_i), \qquad \textbf{(3.13)}$$

where $P(y|x_i', pa_i)$ and $P(pa_i)$ represent preintervention probabilities.

Compute $P(y|\hat{x}_i')$???



Identifiability

Let $Q(M)$ be any computable quantity of a model M. We say that Q is identifiable in a class $\boldsymbol{M}$ of models if, for any pairs of models M_1 and M_2 from $\boldsymbol{M}$, $Q(M_1) = Q(M_2)$ whenever $P_{M_1}(v) = P_{M_2}(v)$.

If our observations are limited, and permit only a partial set F_M of features (of $P_M(v)$) to be estimated, we define Q to be identifiable from F_M if $Q(M_1) = Q(M_2)$ whenever $F_{M_1} = F_{M_2}$.

Causal-effect identifiability

The **causal effect** of X on Y is said to be **identifiable** from a graph G if the quantity $P(y|\hat{x})$ can be computed uniquely from any positive probability of the observed variables—that is, if $P_{M_1}(y|\hat{x}) = P_{M_2}(y|\hat{x})$ for every pair of models M_1 and M_2 with $P_{M_1}(v) = P_{M_2}(v) > 0$ and $G(M_1) = G(M_2) = G$.

$$P(y|\hat{x}'_i) = \sum_{pa_i} P(y|x'_i, pa_i)P(pa_i), \tag{3.13}$$

Given a causal diagram G of any Markovian model in which a subset V of variables are measured, the causal effect $P(y|\hat{x})$ is identifiable whenever $\{X \cup Y \cup PA_X\} \subseteq V$, that is, whenever X, Y, and all parents of variables in X are measured. The expression of $P(y|\hat{x})$ is then obtained by adjusting for PA_x, as in (3.13).

Given the causal diagram G of any Markovian model in which all variables are measured, the causal effect $P(y|\hat{x})$ is identifiable for every two subsets of variables X and Y and is obtained from the truncated factorization of (3.14).

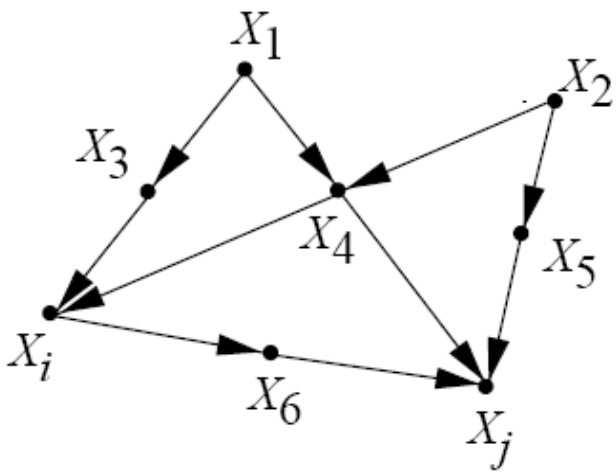
$$P(x_1, \ldots, x_n|\hat{s}) = \begin{cases} \prod_{i|X_i \notin S} P(x_i|pa_i) & \text{for } x_1, \ldots, x_n \text{ consistent with } s, \\ 0 & \text{otherwise.} \end{cases} \tag{3.14}$$

Controlling confounding bias – Back-Door Criterion

A set of variables Z satisfies the **back-door** criterion relative to an ordered pair of variables (X_i, X_j) in a DAG G if:

(i) no node in Z is a descendant of X_i; and

(ii) Z blocks every path between X_i and X_j that contains an arrow into X_i.



Back-Door Adjustment

If a set of variables Z satisfies the back-door criterion relative to (X, Y), then the causal effect of X on Y is identifiable and is given by the formula

$$P(y|\hat{x}) = \sum_z P(y|x, z)P(z).$$

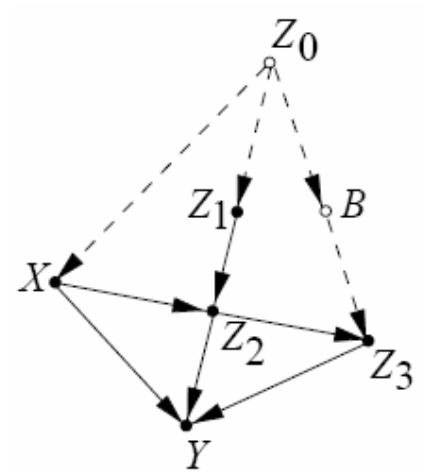
Controlling confounding bias – Front-Door Criterion

A set of variables Z is said to satisfy the **front-door** criterion relative to an ordered pair of variables (X, Y) if:

(i) Z intercepts all directed paths from X to Y;

(ii) there is no back-door path from X to Z; and

(iii) all back-door paths from Z to Y are blocked by X.



Front-Door Adjustment

If Z satisfies the front-door criterion relative to (X, Y) and if $P(x, z) > 0$, then the causal effect of X on Y is identifiable and is given by the formula

$$P(y|\hat{x}) = \sum_z P(z|x) \sum_{x'} P(y|x', z) P(x').$$

Intervention calculus – Rules of *d* Calculus

Let G be the directed acyclic graph associated with a causal model as defined in (3.2), and let $P(\cdot)$ stand for the probability distribution induced by that model. For any disjoint subsets of variables X, Y, Z, and W we have the following rules.

Rule 1 (Insertion/deletion of observations) :

$$P(y|\hat{x}, z, w) = P(y|\hat{x}, w) \quad \text{if} \quad (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}}}.$$

Rule 2 (Action/observation exchange) :

$$P(y|\hat{x}, \hat{z}, w) = P(y|\hat{x}, z, w) \quad \text{if} \quad (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}\underline{Z}}}.$$

Rule 3 (Insertion/deletion of actions) :

$$P(y|\hat{x}, \hat{z}, w) = P(y|\hat{x}, w) \quad \text{if} \quad (Y \perp\!\!\!\perp Z|X,\ W)_{G_{\overline{X},\ \overline{Z(W)}}},$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Implications of *d*-Calculus

A causal effect $q = P(y_1, ..., y_k | \hat{x}_1, ..., \hat{x}_m)$ is identifiable in a model characterized by a graph G if there exists a finite sequence of transformations, each conforming to one of the inference rules in Theorem 3.4.1, that reduces q into a standard (i.e. "hat"-free) probability expression involving observed quantities.

Notations of *do* Calculus

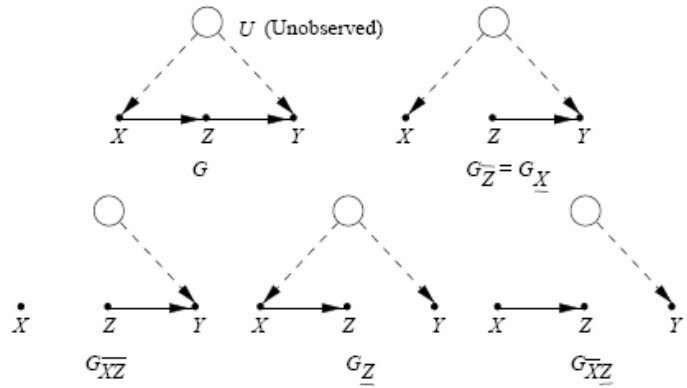


Figure 3.6: Subgraphs of G used in the derivation of causal effects.

$G_{\overline{X}}$ remove arrows pointing to X

$G_{\underline{X}}$ remove arrows emanating from X

$G_{\overline{X}\underline{Z}}$ remove ears of X and legs of Z

$$P(y|\widehat{x}, z) \triangleq \frac{P(y, z|\widehat{x})}{P(z|\widehat{x})}$$

Graphical tests of Identifiability

Identifying models

Non-identifying models

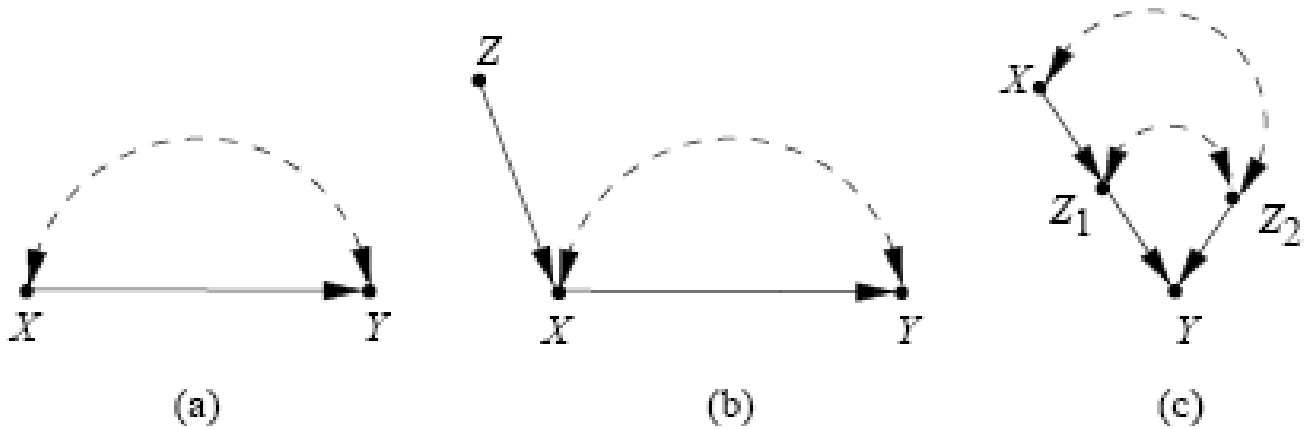


Figure 3.7: (a) A bow pattern: a confounding arc embracing a causal link $X \to Y$, thus preventing the identification of $P(y|\hat{x})$ even in the presence of an instrumental variable Z, as in (b). (c) A bowless graph that still prohibits the identification of $P(y|\hat{x})$.

Identifying models

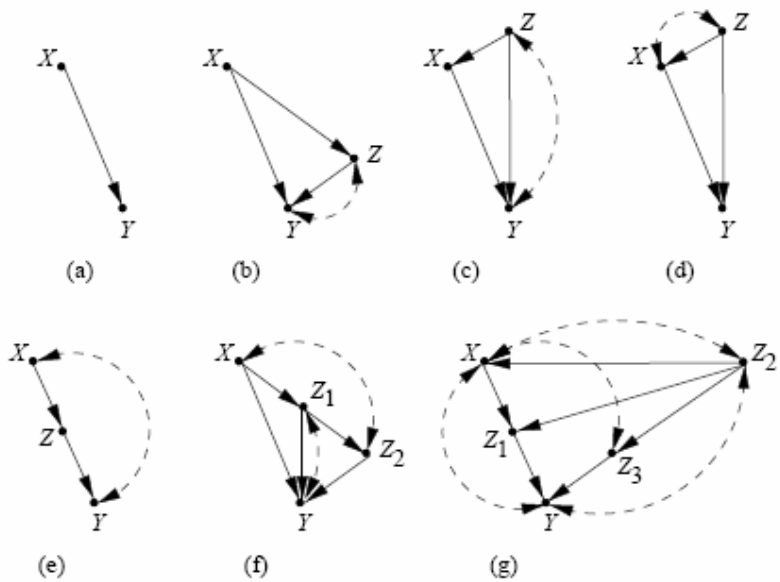


Figure 3.8: Typical models in which the effect of X on Y is identifiable. Dashed arcs represent confounding paths, and Z represents observed covariates.

Non-identifying models

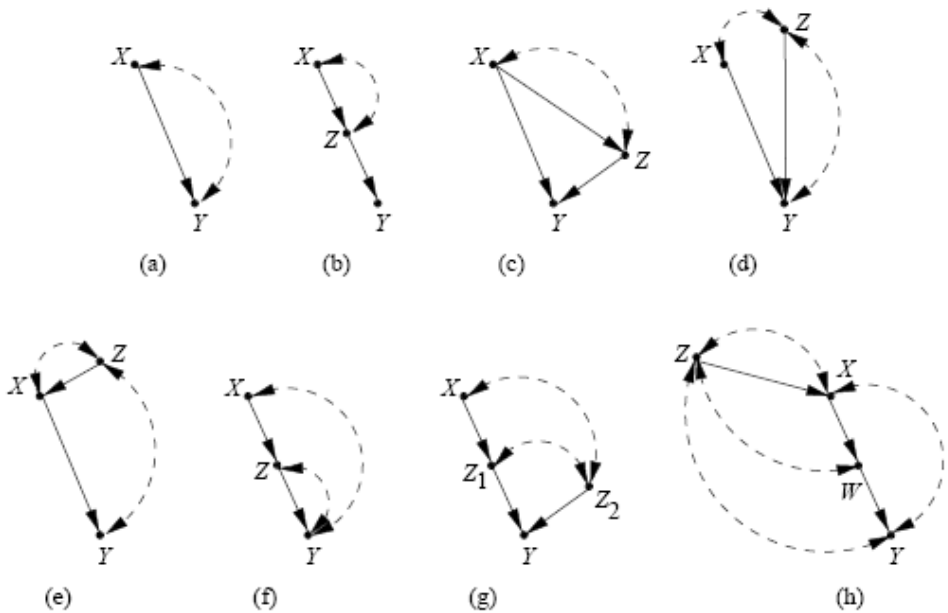
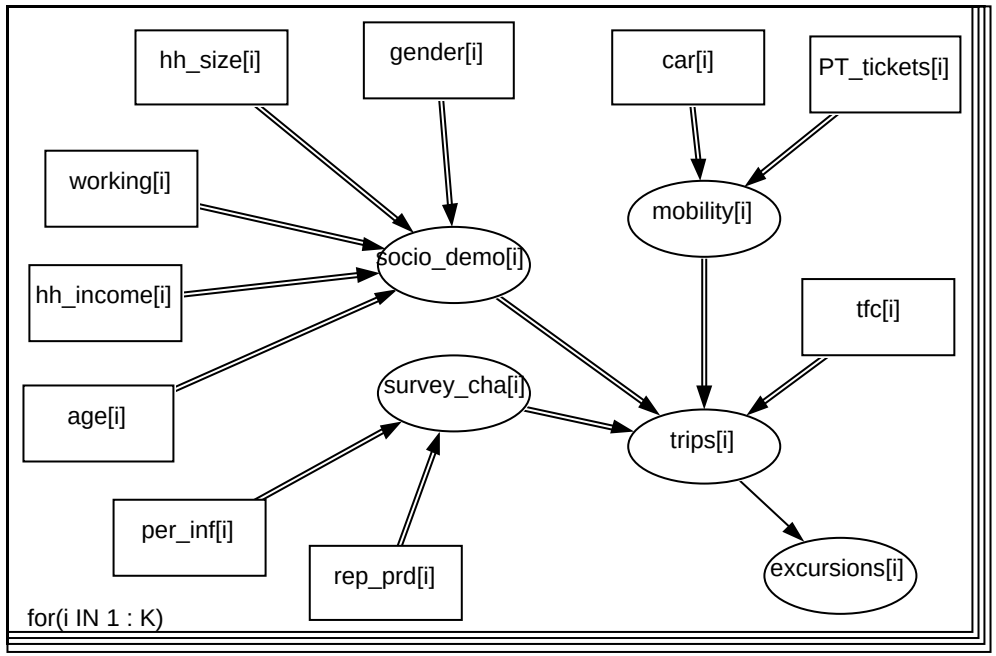


Figure 3.9: Typical models in which $P(y|\hat{x})$ is not identifiable.

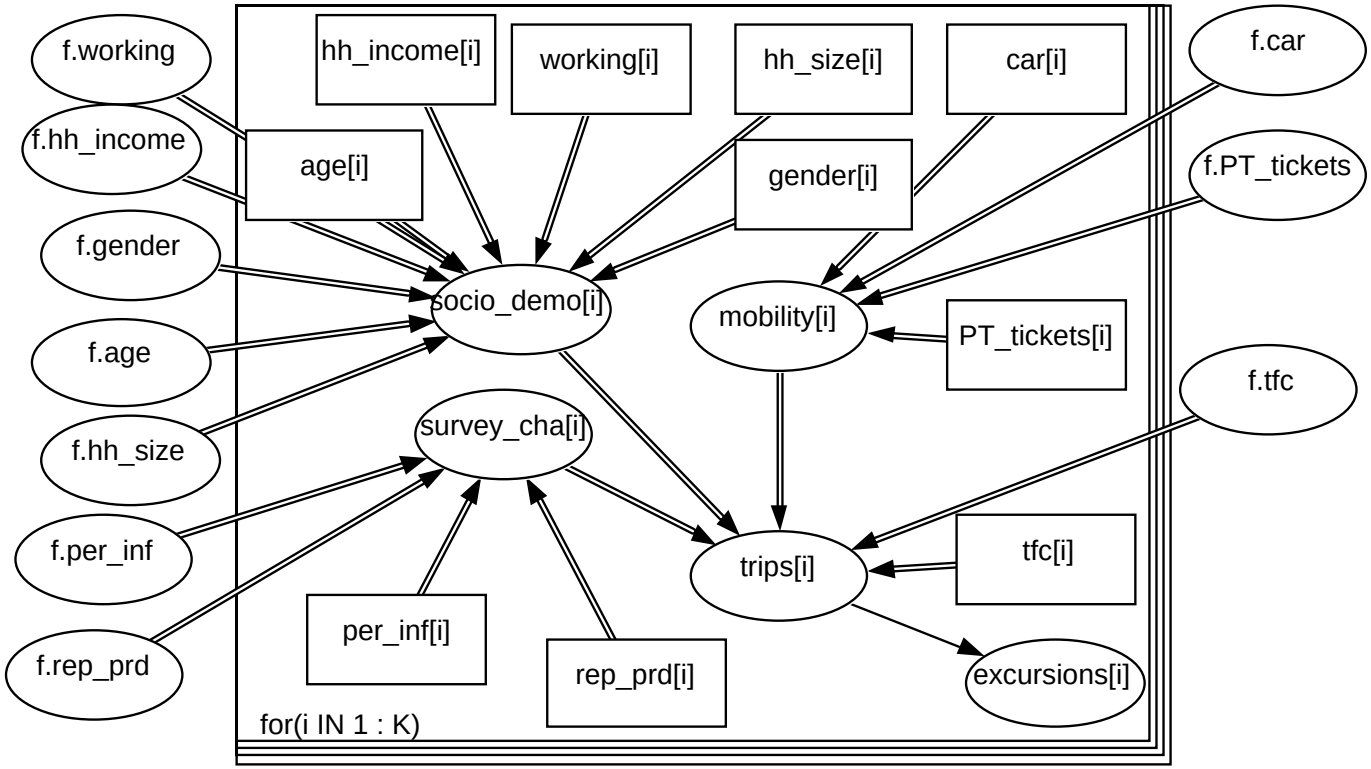
Graphical model in transport domain (WinBUGS)



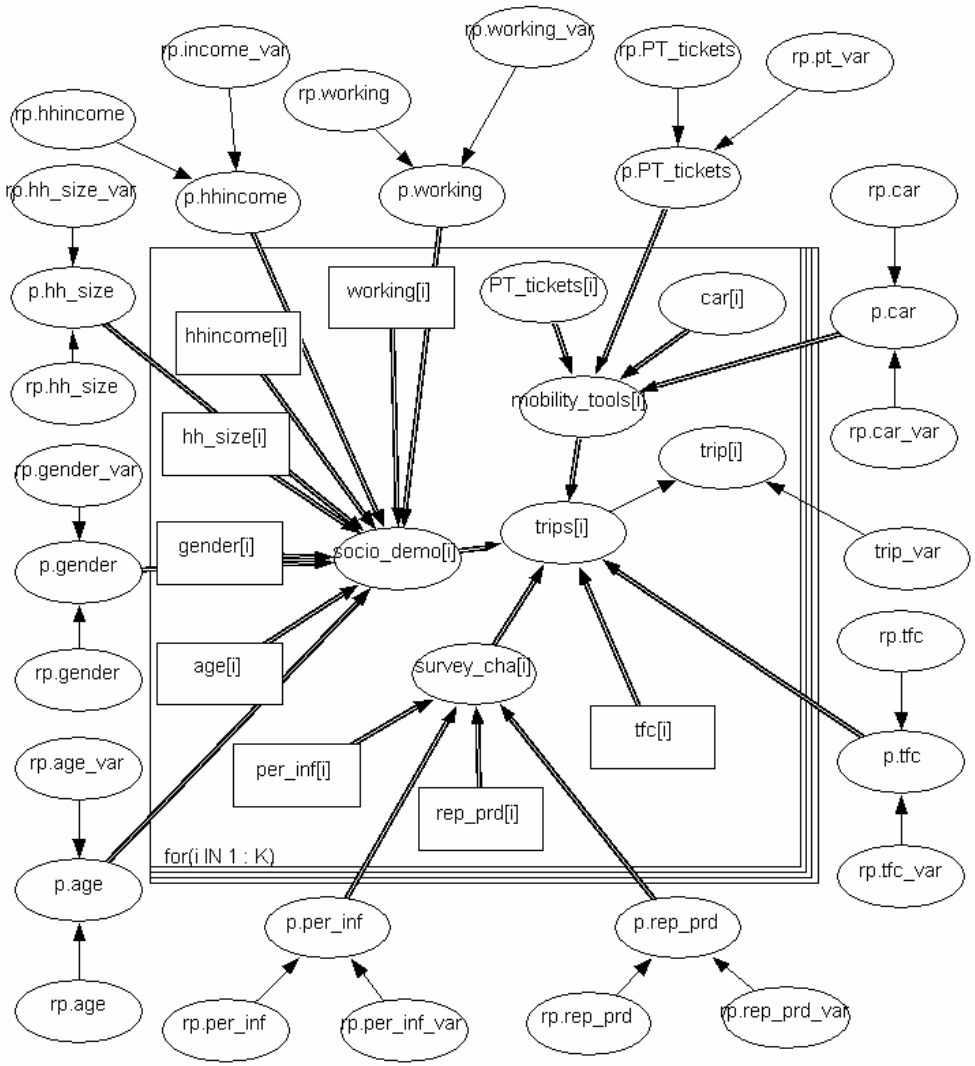
Excursions ~ [*f*(*Socio-demographics*) + *f*(*Mobility tool ownership*) + *f*(*Land-use type*) + *f*(*Survey characteristics*)]

Is it a causal diagram??

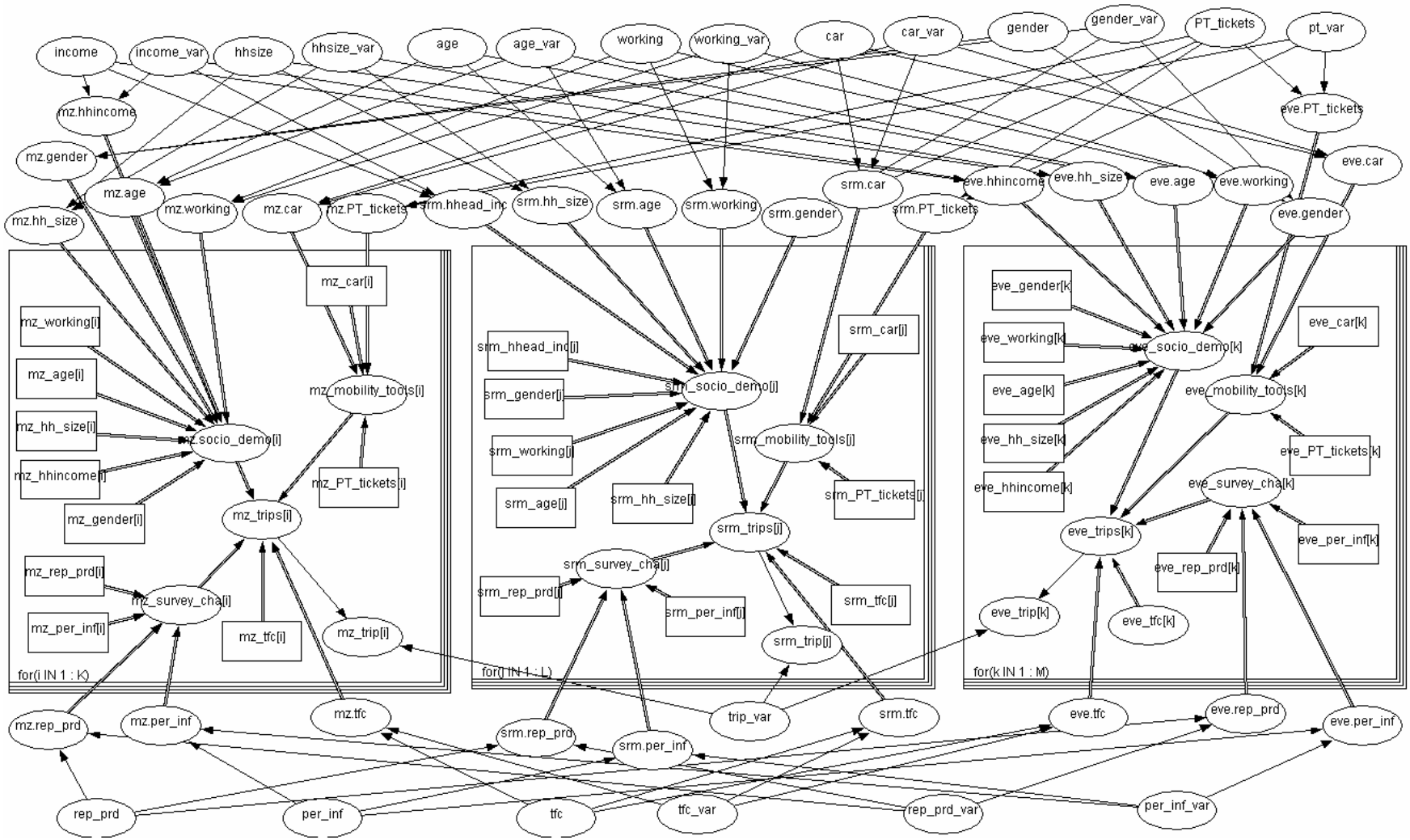
Simple regression model



Random-effect regression model



Random effect regression model for multi-source data



Adequacy of the model

Model complexity

Model with lease number of free (in-effective) parameters is the least complex

Model fit

Model with maximum likelyhood function (Deviance information criterion – DIC) fits the best