"BOOTSTRAP MATLAB TOOLBOX"

Version 2.0 (May 1998)

Software Reference Manual

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Contents

Interacitve Examples

```
out=bootrsp(in,B)

Bootstrap  resampling  procedure.

  Inputs:
     in - input data
      B - number of bootstrap resamples (default B=1)
  Outputs:
    out - B bootstrap resamples of the input data

For a vector input data of size [N,1], the  resampling
procedure produces a matrix of size [N,B] with columns
being resamples of the input vector.

For a matrix input data of size  [N,M], the resampling
procedure produces a 3D matrix of  size  [N,M,B]  with
out(:,:,i), i = 1,...,B, being a resample of the input
matrix.
```

Example

```
>>  x=1:5;
>>  out=bootrsp(x,10)

    out =

        2     5     5     5     2     3     2     3     4     1
        5     5     3     4     1     5     4     1     1     5
        5     3     1     3     1     5     3     5     3     1
        1     4     5     3     3     2     5     3     5     4
        3     3     5     3     5     1     5     3     5     4
```

See Also : bootrsp2, jackrsp

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
[out1,out2]=bootrsp2(in1,in2,B)

Bootstrap  resampling  procedure for bivariate data.

  Inputs:
    in1 - input data (first variate)
    in2 - input data (second variate). If in2 is not
          provided the function runs bootrsp.m by default.
      B - number of bootstrap resamples (default B=1)
  Outputs
   out1 - B bootstrap resamples of the first variate
   out2 - B bootstrap resamples of the second variate

For a vector input data of size [N,1], the  resampling
procedure produces a matrix of size [N,B] with columns
being resamples of the input vector.
```

Example

```
>>  x=1:5;
>>  y=5:9;

>>  [out1,out2]=bootrsp2(x,y,10)

    out1 =

        5     4     1     1     3     3     1     5     3     1
        4     2     1     4     5     5     2     1     5     2
        2     2     5     2     3     3     3     2     5     5
        2     2     4     3     2     4     2     4     5     2
        4     5     1     5     4     4     1     2     5     5

    out2 =

        9     8     5     5     7     7     5     9     7     5
        8     6     5     8     9     9     6     5     9     6
        6     6     9     6     7     7     7     6     9     9
        6     6     8     7     6     8     6     8     9     6
        8     9     5     9     8     8     5     6     9     9
```

See Also : bootrsp, jackrsp

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 D=boottest(x,statfun,v_0,type,alpha,B1,B2,PAR1,...)

 Hypothesis test for a characteristic (parameter) 'v'
 of an unknown distribution  based on the bootstrap
 resampling procedure and pivoted test statistics

Inputs:
      x - input vector data
statfun - the estimator of the parameter given as a Matlab function
   v_0  - the value of 'v' under the null hypothesis
   type - the type of hypothesis test.

          For type=1:   H: v=v_0   against K: v~=v_0
           (two-sided hypothesis test)
          For type=2:   H: v<=v_0  against K: v>v_0
           (one-sided hypothesis test)
          For type=3:   H: v>=v_0  against K: v<v_0
           (one-sided hypothesis test)
          (default type=1)
 alpha  - the level of the test (default alpha=0.05)
     B1 - number of bootstrap resamplings
          (default B1=99)
     B2 - number of bootstrap resamplings for variance
          estimation (nested bootstrap)
          (default B2=25)
PAR1,... - other parameters than x to be passed to statfun

Outputs:
     D - The output of the test.
          D=0: retain the null hypothesis
          D=1: reject the null hypothesis
```

Example

```
>>  x=randn(1,100);
>>  D=boottest(x,'trimmean',0,1,0.05,199,25,20)
    D =

       0

>>  D=boottest(x,'trimmean',0.5,1,0.05,199,25,20)
    D =

       1
```

See Also : boottestnp, boottestvs

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 D=boottestnp(x,statfun,v_0,type,alpha,B,PAR1,...)

 Hypothesis test for a characteristic (parameter) 'v'
 of an unknown distribution  based on the bootstrap
 resampling procedure and unpivoted test statistics

Inputs:
      x - input vector data
statfun - the estimator of the parameter given as a Matlab function
   v_0  - the value of vartheta under the null hypothesis
   type - the type of hypothesis test.

          For type=1:   H: v=v_0   against K: v~=v_0
           (two-sided hypothesis test)
          For type=2:   H: v<=v_0  against K: v>v_0
           (one-sided hypothesis test)
          For type=3:   H: v>=v_0  against K: v<v_0
           (one-sided hypothesis test)
          (default type=1)
 alpha  - determines the level of the test
          (default alpha=0.05)
      B - number of bootstrap resamplings
          (default B1=99)
PAR1,... - other parameters than x to be passed to statfun

Outputs:
      D - The output of the test.
          D=0: retain the null hypothesis
          D=1: reject the null hypothesis
```

Example

```
>> w=randn(1,100);
>> x=randn(1,100)+w;
>> y=randn(1,100)+w;
>> D=boottestnp(x,'correl',0.5,1,0.05,199,y)

   D =

      0

>> D=boottestnp(x,'correl',0.6,1,0.05,199,y)
```

```
D =

     1
```

where correl.m function returns the first element of the output produced by the corrcoef.m.

See Also : boottest, boottestvs

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 D=boottestvs(x,statfun,v_0,type,alpha,B1,B2,B3,PAR1,...)

 Hypothesis test for a characteristic (parameter) 'v'
 of an unknown distribution  based on the bootstrap
 resampling procedure and variance stabilisation (VS).

Inputs:
      x - input vector data
statfun - the estimator of the parameter given as a Matlab function
   v_0  - the value of vartheta under the null hypothesis
   type - the type of hypothesis test.

          For type=1:   H: v=v_0   against K: v~=v_0
           (two-sided hypothesis test)
          For type=2:   H: v<=v_0  against K: v>v_0
           (one-sided hypothesis test)
          For type=3:   H: v>=v_0  against K: v<v_0
           (one-sided hypothesis test)
          (default type=1)
 alpha  - determines the level of the test
          (default alpha=0.05)
    B1  - numbers of bootstrap resamplings for VS
          (default B1=100)
    B2  - numbers of bootstrap resamplings for VS
          (default B2=25)
    B3  - number of bootstrap resamplings
          (default B3=99)
PAR1,... - other parameters than x to be passed to statfun

Outputs:
      D - The output of the test.
          D=0: retain the null hypothesis
          D=1: reject the null hypothesis
```

Example

```
>> x=normrnd(5,3,1,100);
>> D = boottestvs(x,'std',3,1,0.05,100,25,199)

D =

    0
```

```
>> D = boottestvs(x,'std',3.5,1,0.05,100,25,199)

D =

     1
```

See Also : boottest, boottestnp

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
    [est,estvar]=bpestcir(X,estfun,L1,M1,Q1,L2,M2,Q2,B,PAR1,...)

    The program calculates the estimate and the variance
    of an estimator of a parameter from the input vector X.
    The algorithm is based  on a circular block bootstrap
    and is suitable when the data is weakly correlated.

 Inputs:
       x - input vector data
  estfun - the estimator of the parameter given as a Matlab function
      L1 - number of elements in the first block (see "segmcirc.m")
      M1 - shift size in the first block
      Q1 - number of segments in the first block
      L2 - number of elements in the second block (see "segmcirc.m")
      M2 - shift size in the second block
      Q2 - number of segments in the second block
       B - number of bootstrap resamplings (default B=99)
PAR1,... - other parameters than x to be passed to estfun

 Outputs:
     est - estimate of the parameter
  estvar - variance of the estimator
```

Example

```
>> x=normrnd(5,3,1,1000);
>> [est,estvar]=bpestcir(x,'mean',50,50,50,10,10,10)

est =

    4.9916

estvar =

    0.0193

>> [est,estvar]=bpestcir(x,'std',50,50,50,10,10,10)

est =

    2.9961

estvar =

    0.0070
```

See Also : bpestdb

References

1. Politis, N.P. and Romano, J.P. Bootstrap Confidence Bands for Spectra and Cross-Spectra. *IEEE Transactions on Signal Processing*, Vol. 40, No. 5, 1992.

2. Zhang, Y. *et. al.* Bootstrapping Techniques in the Estimation of Higher Order Cumulants from Short Data Records. In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing, ICASSP-93*, Vol. IV, pp. 200-203.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
      [est,estvar]=bpestdb(X,estfun,L1,M1,L2,M2,B,PAR1,...)

      The program calculates the estimate and the variance
      of an estimator of a parameter from the input vector X.
      The algorithm is based  on a double block bootstrap
      and is suitable when the data is weakly correlated.

   Inputs:
         x - input vector data
    estfun - the estimator of the parameter given as a Matlab function
        L1 - number of elements in the first block (see "segments.m")
        M1 - shift size in the first block
        L2 - number of elements in the second block (see "segments.m")
        M2 - shift size in the second block
         B - number of bootstrap resamplings (default B=99)
  PAR1,... - other parameters than x to be passed to estfun

   Outputs:
       est - estimate of the parameter
    estvar - variance of the estimator
```

Example

```
>> x=normrnd(5,3,1,1000);
>> [est,estvar]=bpestdb(x,'mean',50,50,10,10)

est =

    4.9352

estvar =

    0.0018

>> [est,estvar]=bpestdb(x,'std',50,50,10,10)

est =

    3.0061

estvar =

    0.0820
```

See Also : bpestcir

References

1. Politis, N.P. and Romano, J.P. Bootstrap Confidence Bands for Spectra and Cross-Spectra. *IEEE Transactions on Signal Processing*, Vol. 40, No. 5, 1992.

2. Zhang, Y. *et. al.* Bootstrapping Techniques in the Estimation of Higher Order Cumulants from Short Data Records. In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing, ICASSP-93*, Vol. IV, pp. 200-203.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 [Lo,Up]=confint(x,statfun,alpha,B1,B2,PAR1,...)

 Confidence interval of the estimator of a parameter
 based on the bootstrap percentile-t method

Inputs:
      x - input vector data
statfun - the estimator of the parameter given as a Matlab function
 alpha  - level of significance (default alpha=0.05)
     B1 - number of bootstrap resamplings (default B1=199)
     B2 - number of bootstrap resamplings for variance
          estimation (nested bootstrap) (default B2=25)
PAR1,... - other parameters than x to be passed to statfun

Outputs:
    Lo - The lower bound
    Up - The upper bound
```

Example

```
>> x=exprnd(0.5,1,100);
>> [mu,bands]=expfit(x,0.05)

mu =

    0.4965

bands =

    0.4039
    0.5984

>> [Lo,Up] = confint(x,'mean')

Lo =

    0.3946

Up =

    0.6111
```

See Also : confintp, confinth

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Hall, P. Theoretical Comparison of Bootstrap Confidence Intervals. *The Annals of Statistics*, Vol 16, No. 3, pp. 927-953, 1988.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
  [Lo,Up]=confintp(x,statfun,alpha,B1,PAR1,...)

  Confidence interval of the estimator of a parameter
  based on the bootstrap percentile method

 Inputs:
       x - input vector data
 statfun - the estimator of the parameter given as a Matlab function
  alpha  - level of significance (default alpha=0.05)
      B1 - number of bootstrap resamplings (default B1=199)
PAR1,... - other parameters than x to be passed to statfun

 Outputs:
     Lo - The lower bound
     Up - The upper bound
```

Example

```
>> x=exprnd(0.5,1,100);
>> [mu,bands]=expfit(x,0.05)

mu =

    0.4965

bands =

    0.4039
    0.5984

>> [Lo,Up] = confintp(x,'mean')

Lo =

    0.4139

Up =

    0.5923
```

See Also : confint, confinth

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Hall, P. Theoretical Comparison of Bootstrap Confidence Intervals. *The Annals of Statistics*, Vol 16, No. 3, pp. 927-953, 1988.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 [Lo,Up]=confinh(x,statfun,alpha,B1,PAR1,...)

 Confidence interval of the estimator of a parameter
 based on the bootstrap hybrid method

Inputs:
      x - input vector data
statfun - the estimator of the parameter given as a Matlab function
 alpha  - level of significance (default alpha=0.05)
     B1 - number of bootstrap resamplings (default B1=199)
PAR1,... - other parameters than x to be passed to statfun

Outputs:
     Lo - The lower bound
     Up - The upper bound
```

Example

```
>> x=exprnd(0.5,1,100);
>> [mu,bands]=expfit(x,0.05)

mu =

    0.4965

bands =

    0.4039
    0.5984

>> [Lo,Up] = confinth(x,'mean')

Lo =

    0.3970

Up =

    0.5923
```

See Also : confint, confintp

References

1. Efron, B.and Tibshirani, R. *An Introduction to the Bootstrap*, Chapman and Hall, 1993.

2. Hall, P. Theoretical Comparison of Bootstrap Confidence Intervals. *The Annals of Statistics*, Vol 16, No. 3, pp. 927-953, 1988.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
 [est,estall]=jackest(x,estfun,h,PAR1,...)

  Parameter estimation based on the "Jackknife" procedure

  Inputs:
        x - input vector data
   estfun - the estimator of the parameter given as a Matlab function
        h - number of elements in a block that is to be deleted.
            see jackrsp.m (defult h=1)
 PAR1,... - other parameters than x to be passed to estfun

  Outputs:
      est - the jackknifed estimate
   estall - the estimate based on the whole sample
```

Example

```
>> x=normrnd(5,3,1,20);
>> [est,estall]=jackest(x,'trimmean',1,20)

est =

    4.7086

estall =

    4.6909
```

See Also : jackrsp

References

1. Efron, B. Bootstrap Methods. Another Look at the Jackknife. *The Annals of Statistics*, Vol. 7, pp. 1-26, 1979.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
y=jackrsp(x,h)

The procedure known as a "Jackknife" forms a matrix of size
(g-1)*h by g from the input vector x of length g*h. The
input vector is first divided into "g" blocks of size  "h".
Each column of the matrix is formed  by deleting a block
from the input. The standard version of the Jackknife is
when h=1.

Inputs:
     x - input vector data
     h - number of elements in a block that is to be deleted
         (default h=1)
Output:
     y - the output matrix of the data
```

Example

```
>> x=1:6;
>> y=jackrsp(x,1)

y =

     2     1     1     1     1     1
     3     3     2     2     2     2
     4     4     4     3     3     3
     5     5     5     5     4     4
     6     6     6     6     6     5

>> y=jackrsp(x,2)

y =

     3     1     1
     4     2     2
     5     5     3
     6     6     4
```

See Also : jackest

References

1. Efron, B. Bootstrap Methods. Another Look at the Jackknife. *The Annals of Statistics*, Vol. 7, pp. 1-26, 1979.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
y=segmcirc(X,L,M,Q)

Given the data samples X=(x_1,x_2,...,x_N),
the program obtains Q overlapping (M<L) or
non-overlapping (M>=L) segments, each of L samples
in the form of a matrix "y" of L rows and Q columns.
The data X_i is "wrapped" around in a circle, that is,
define (for i>N) X_i=X_iN, where iN=i(mod N).
       _______________
 .....|_______L_______| .....
 .....|____M____|_______L_______| .....
 .....|___ M ___|___ M ___|______ L ______| .....

The procedure is used for the circular block bootstrap.

Inputs:
     X - input vector data
     L - number of elements in a segment
     M - shift size (i.e. L-M is the size of overlap)
     Q - number of desired segments
Output:
     y - the output matrix of the data
```

See Also : segments

References

1. Politis, N.P. and Romano, J.P. Bootstrap Confidence Bands for Spectra and Cross-Spectra. *IEEE Transactions on Signal Processing*, Vol. 40, No. 5, 1992.

2. Zhang, Y. *et. al.* Bootstrapping Techniques in the Estimation of Higher Order Cumulants from Short Data Records. In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing, ICASSP-93*, Vol. IV, pp. 200-203.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
[y,Q]=segments(X,L,M)

Given the data samples X=(x_1,x_2,...,x_N),
the program obtains Q overlapping (M<L) or
non-overlapping (M>=L) segments, each of L samples
in the form of a matrix "y" of L rows and Q columns.
  ______________
 |______ L ______| .....
 |___ M ___|______ L ______| .....
 |___ M ___|___ M ___|______ L ______| .....

The procedure is used for the block of blocks bootstrap.

Inputs:
     X - input vector data
     L - number of elements in a segment
     M - shift size (i.e. L-M is the size of overlap)
Output:
     y - the output matrix of the data
     Q - number of output segments
```

See Also : segmcirc

References

1. Politis, N.P. and Romano, J.P. Bootstrap Confidence Bands for Spectra and Cross-Spectra. *IEEE Transactions on Signal Processing*, Vol. 40, No. 5, 1992.

2. Zhang, Y. *et. al.* Bootstrapping Techniques in the Estimation of Higher Order Cumulants from Short Data Records. In *Proceedings of the International Conference on Acoustics, Speech and Signal Processing, ICASSP-93*, Vol. IV, pp. 200-203.

3. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

4. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.

```
        [x_sort, y_sort, y_sm] = smooth(x, y, w);

        A running line smoother that fits the data by linear
        least squares. Used to compute the variance stabilising
        transformation.

  Inputs:
        x - one or more columns of covariates
        y - one column of response for each column of covariate
        w - span, proportion of data in symmetric centred window

 Outputs:
   x_sort - sorted columns of x
   y_sort - values of y associated with x_sort
     y_sm - smoothed version of y

Note: If inputs are row vectors, operation is carried out row-wise.
```

References

1. Hastie, T.J. and Tibshirani, R.J. *Generalised additive models.* Chapman and Hall, 1990.

2. Zoubir, A.M. Bootstrap Theory and Applications. *Proceedings of the SPIE 1993 Conference on Advanced Signal Processing Algorithms, Architectures and Implementations*, pp. 216-235, San Diego, July 1993.

3. Zoubir, A.M. and Boashash, B. The Bootstrap: Signal Processing Applications. *IEEE Transactions on Signal Processing*, Vol. 15, No. 1, pp. 55-76, 1998.